Strategic Planning
Aggregate Planning

⇒ Which plant should produce what?
⇒ Where to locate a new plant/facility?
⇒ How much should a plant be able to produce?
⇒ How many customers should a facility be able to serve?
⇒ How much should a plant produce each month?

1. THE PLANNING DECISIONS
1.1 THE PRODUCT SIDE
1.2 THE CORPORATE SIDE
1.3 HIERARCHICAL PRODUCTION PLANNING

2. STRATEGIC PLANNING
2.1 STRATEGIC QUESTIONS
2.2 QUALIFIERS AND ORDER WINNERS
2.3 MANUFACTURING STRATEGY
2.4 CAPACITY REQUIREMENTS
2.5 FACILITY LOCATION

3. AGGREGATE PLANNING
3.1 EXAMPLE: RADIATOR CALORIX
3.2 OBJECTIVES & VARIABLES
3.3 GRAPHICAL METHOD
3.4 LP FORMULATION
3.5 LINEAR DECISION RULE
3.6 MODELING MANAGEMENT BEHAVIOR
3.7 CONCLUSIONS

4. DISAGGREGATING THE PLAN: MPS
4.1 SCOPE
4.2 EXAMPLES

1. The planning decisions

1.1 The Product Side

In order to understand the planning process in a company, let us first consider all the steps from the design of a product up to its manufacturing. The birth generally follows a 3-step procedure (see the previous section on products and processes). First the idea of the product is born and supported. Second, the product is designed and specified. Third, the process by which the product will (or could) be manufactured is selected.

### Process Flow Diagram

- **Consumer needs** ➞ **Idea generation** ➞ **Selection**
  - Idea generation ➞ **Market analysis**
  - Market analysis ➞ **Product development** ➞ **Choose features and set the final specifications**
  - Product development ➞ **Evaluation of technologies and methods** ➞ **Process Selection** ➞ **Choose specific equipment and process flow**

These different decisions have been described as being taken sequentially. This was for clarity reasons. In reality, there is some feedback between the different decisions and some constraints apply through several layers of decisions. For example:

**The market constraints:**
- the product development;
- the process selection;
- the planning activities;

When the process selection has been performed, we know what to produce and how. There are still many questions to be answered:
  - where will the product be manufactured?
  - will the product be manufactured with others?
  - how will the customers be served: from stock or on demand?
  - at what time and how many units will be manufactured?

This is the world of strategic and capacity planning we study in this section.
1.2 The Corporate Side

Here are the successive decision steps by which a company manufactures a product. Roughly, the strategic planning defines which products to manufacture and where. This is a long term decision based on "business forecasting".

The aggregate planning looks at the medium term and selects the best policy to cope with the fluctuation of the global demand during a period of about 12-24 months.

Finally, the production activities and the requirements in terms of material are determined for the short term, that is a few weeks.

For the long term, the decisions are first strategic. Here the products and the target market are selected. It will also be decided where to produce what.

The aggregate planning is based on an aggregate production target per time period (the month usually). It aims at selecting the right combination of work force, of inventory levels and of subcontracting. The resource planning mainly focuses on minor equipment and personnel change.

The MPS (master production schedule) refers to the production objectives, per product and per time period (the week usually) for a term of about 1 to 3 months. The MRP (material requirement planning) refers to the short term. It specifies a planning for the parts: how many parts are required to reach the MPS and when. MPS and MRP will be studied in detail in the next chapter.

Note that the meaning of long/medium/short term varies with the industry sector.
### 1.3 Hierarchical Production Planning

Here is another way of structuring the different planning decisions. It is based on the question: "who decides what for which time period and on the basis of which data ?".

<table>
<thead>
<tr>
<th>Needed Forecast</th>
<th>Decision Process</th>
<th>Decision Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Demand  (\Rightarrow) by product line and by region</td>
<td>Allocates production among plants</td>
<td>⇨ Corporate</td>
</tr>
<tr>
<td>Monthly (\Rightarrow) demand for 15 months by product type</td>
<td>determines seasonal plan by product type</td>
<td>⇨ Plant Manager</td>
</tr>
<tr>
<td>Weekly demand (\Rightarrow) for 5 months by item</td>
<td>determines monthly item production schedules</td>
<td>⇨ Shop Superintendent</td>
</tr>
</tbody>
</table>

The long term plan is defined at the corporate level. These decisions are more strategic. This activity is often referred to as "strategic planning".

The medium or intermediate term plan is defined at the plant level. This activity is called "aggregate planning" because the demand for the different products to be manufactured is aggregated.

In this chapter we will successively deal with the strategic and the aggregate planning.

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2. Strategic Planning

2.1 Strategic Questions

Choosing between an individual, a line or a job shop organization is a difficult question. Neither is better or worse. They just have different features.

Choosing the country in which to manufacture the products is also difficult. Should we select a low wage country located far from the customers or the opposite?

None of these questions has a definite answer. We need a framework that provides us guidelines to answer these questions.

Questions:

- which market segment?
- how to reach it?
- which plant / facility?
- which production policy?
- which production process?

Answers:

need for a framework

This framework will most often be determined by the target market segment. Let us look at an example. If your market segment is "warm noon meals for students", then you should focus on cheap meals served where the students are. An adequate manufacturing strategy would be then to use a "line production organization" for producing many standard meals, to employ low qualified workers and to deliver these meals in facilities close to the students. This is a simple example which illustrates how the market segment influences the production process, the kind of workforce and the facility location.

need for market segment specifications

The example above is a sequential decision process. The market segment defines some manufacturing characteristics. In the real world, some feedback should be encouraged. For example, if your existing restaurant facilities are located far from the student population or if you have only highly qualified workers, you should not aim at the student segment. In this case, you need either to adapt your market segment or your manufacturing characteristics.

In other words, choosing a market segment is not just a marketing decision. It should result from a clear collaboration between all the departments (marketing, manufacturing, R&D, logistics, finance, human resources). The role of the manufacturing department is very clear. It will be in charge of satisfying the customer orders. Therefore, the conditions under which these orders have to be satisfied must be made explicit. The manufacturing department must decide whether it is able to meet these conditions or not.

need for collaboration between departments

How can we organize this collaboration between marketing and manufacturing? One solution is to organize the discussion around the “qualifiers” and the “order winners” proposed by T. Hill
2.2 Qualifiers and Order Winners

The idea is first to translate the attributes of the market segment into qualifiers and order winners. Then, these Q&OW’s will provide the framework for making decisions.

**Attribute:** A feature of the service/product;

The table below shows a long list of attributes. The time to fill an order is an example.

**Qualifier:** An attribute whose value allows me to qualify as a potential supplier;

If all my competitors do deliver to their customers within 24 hours, I must reach this same target. If I deliver within 3 days only, I will most likely not qualify, that is, I will not be in their short list of potential suppliers. 24 hours is the delay I must reach for qualifying.

**Order winner** An attribute whose value allows me to win customer orders;

But I could decide to deliver my product within 6 hours. In this case, I hope to win the orders from all the customers for whom this shorter delay is essential.

<table>
<thead>
<tr>
<th>Corporate Objectives</th>
<th>Marketing Strategy</th>
<th>Qualifiers and O.W</th>
<th>Manufacturing Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The discussion between marketing and manufacturing can crystallize around the selection of these qualifiers and order winners. The start of the loop should be, of course, at the corporate level. Then the loop should bounce between marketing and manufacturing until it stabilizes. A detail of this loop is the following table which has been extracted from T. Hill, *Manufacturing Strategy*, pg. 28.

<table>
<thead>
<tr>
<th>Corporate objectives</th>
<th>Growth; Profit, ROI; Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product and market segments Range Mix Volumes Standardization vs. customization Level of innovation Leader vs. follower</td>
<td>Volumes Cost Quality Delivery speed and reliability Demand increases Product range Design Technical support After-sales support</td>
</tr>
<tr>
<td>Marketing Strategy Qualifiers and Order winners</td>
<td>Manufacturing Strategy</td>
</tr>
</tbody>
</table>
2.3 Manufacturing Strategy

At the strategic level, the debate focuses first on the market segments, then on the qualifiers and order winners used to reach these segments and finally on the most appropriate manufacturing/service strategy to meet these Q&OW.

Here we first review the basic questions of the manufacturing/service strategy. When possible, we also review basic answers to these questions.

Vertical integration

- Make or Buy ?
- Own or Lease ?

⇒ break-even / financial analysis

What should be the span of my production system ? Shall I just assemble the parts or shall I fabricate them too? Shall I organize the distribution of my products or can I outsource this function ?

The Q&OW’s provide some help here. If a short delivery time is an OW, you had better keep a good control on the distribution function.

Focus (Plants within plants: PWP)

No company can excel in all performances (cost, quality, flexibility, service,...). For example, the same factory cannot compete on the basis of cost and quality and, at the same time, be flexible. Clear objectives derived from the Q&OW must thus be chosen.

If contradictory objectives exist, then the plant could be split into parts corresponding to the different segments. This leads to the notion of “plants within plants” with all its advantages (small and focused, dynamic, easy to manage, responsible, ...). The force that opposes such a split is the notion of economies of scale.

If one compares the average size of a plant in Japan and in western countries, huge difference appears. Most Japanese plants employ a rather small number of people. This means that if a new product line is launched, then a new plant is build. The main advantages of being small is the ability to be focussed and responsive to the market.

Location

Where shall I locate the plant in charge of manufacturing a given product? The answer depends on whether I want to be cheap, flexible, fast, etc. It thus depends on my Q&OW’s. Basic cost strategies for the location problem are analyzed hereafter.

Capacity

- Nominal capacity

A plant has always a best operating level, that is a level of production which minimizes the unit cost. Below this level, the average cost increases because overhead cost must be allocated to fewer units. Above this level, overtime and higher defect rates are possible causes for a larger unit cost.

\[
\text{Cost per unit} \downarrow \quad \# \text{ Units}
\]

The definition of a best operating level leads to that of the capacity utilization rate:

- Capacity Utilization Rate = Used / Nominal
The Q&OW’s define both the nominal capacity and the capability you must provide for reacting to demand increases.

- **Economies of scale**

Economies of scale result from increased efficiencies and reduced fixed costs. A plant designed for producing 200 units should produce the units at a cost lower than a plant designed for producing 100 units only. Diseconomies of scale could exist too (services).

![Graph showing Economies of Scale](image)

The notions of best operating level and of economy of scales should not be mixed up. If you design a plant for producing 100 units a day, the lowest unit cost will be reached when you produce about 100 units a day. If you produce more, then the unit cost will be higher. This does not result from diseconomies of scale, but from the fact that one does not operate the plant at its best operating level. Economies of scale are possible only if they have been planned.

- **Economies due to experience**

Each time the cumulative production doubles, the cost decreases by a specific amount. This is similar to the learning curve used to estimate the time an operation takes. The economy of scale and the experience advantage can be sought simultaneously.

- **Flexibility: economy of scope**

The aim here is an economy of scope with processes and/or workers who can be used for different products.

**Production Policy**

There are 3 main policies. The choice depends on your Q&OW’s.

- **make-to-order**

The product will be built after the order has been placed. This is used when the product is adapted to the individual customer specifications. The study of a market, the building of an original house and the preparation of a meal are examples of products generally made-to-order.

- **assemble-to-order**

The product is assembled after the order has been placed. In this case, basic modules are manufactured and stored. When an order comes, the final assembly is performed according to the detailed specification of the order. Compared to the previous system, ATO allows a shorter customer lead time. Cars and ice-creams are assembled-to-order.

- **make-to-stock**

The products are manufactured and stored. The customers will be served from stock. Breads and appliances are made-to-stock.

**Process Choice**

Which kind of process organizations will be used: individual, line, or job-shop ? The choice depends on the fit between the general performances of each such organization and your current Q&OW’s.
2.4 Capacity Requirements

When estimating the required production capacity, a four step procedure can be used.

1. Demand Forecasting
   - What will the sales be for each product line and for each region in the coming years?
   - Which service level will be offered?

The first step consists in getting sales forecasts for each product line for, let us say, the next 5 years. If you cannot expect the customers or the products to travel over long distances, your forecasts must be detailed per region. Typical examples are department stores, restaurants, bakeries and schools. On the other hand, if your product can be quickly and cheaply carried over long distances, you could decide to manufacture the products in a single place for the whole market. In this case the global demand is sufficient.

2. Required capacity
   - What capacity is required?

This is just a translation of the forecasts into requirements for the resource you are planning (machine capacity, labor hours, ...). Note that the level of service influences this translation. For example, if the delivery time can be very long, then it is possible to smooth the peak demand over the neighboring periods. If the delivery time must be short, then I need the production capacity to meet the peak demand. We should thus keep the Q&OW’s in mind during the whole process of planning the capacity.

3. Available capacity
   - What capacity is available and where?

Here you list the production capacity which is available and where it is available. In other words, you list here what could be done, where and at what cost.

4. Decision
   - Which sales will be served from which plants?

At this point, strategic decisions could imply the move of a plant to a new location or changes in the production lines or in the production styles of existing plants. The problem of selecting the location for a plant or a shop is analyzed in the next pages.

   ⇒ Facility Location Problem

When the required capacity does not exist (insufficient or inadequate capacity), then investments are necessary.

   - Which existing plants will be enlarged?
   - Which new plants will be built?

When different scenarios for the sales evolution are to be considered, decision trees could provide crucial help.

   ⇒ Decision Tree

Such decision trees analyze various possible scenarios and aim to determine the solution with the highest expected return.
2.5 Facility Location

When it is decided that a plant should be built, the next question is “where to locate it”. This is the plant facility location problem. The same problem arises for a service outlet. The objective for choosing a location must refer to the Q&OW’s.

Objective: meet the Q&OW’s

minimize the costs: - supply
- production
- delivery

The costs always remain a performance measure to be minimized. The supply costs are those generated for getting (buying and carrying) the raw materials to the plant doors. The production costs are directly related to the plant (ground, building, equipment, subcontractors), to the workers (qualification, availability, local wages) and to the environment (public services, political support, social support, taxes). The delivery costs are those generated for bringing the finished goods to the customers.

Here below we propose several techniques. The first technique focuses on the transportation costs only. It can be used to minimize either the delivery or the supply costs. When such costs are high, it can be used to determine the best area for locating the plant or the shop. The second technique is more general (and also more heuristic). It aims at ranking different solutions on the basis of qualitative and quantitative factors. This technique is very useful to shorten the list of candidates.
Technique: 1. Transport cost minimization

The aim of this technique is to locate the plant (or the service outlet) in the "middle" of its customers. The technique proceeds as follows.

1. define where products have to be transported \((a_i, b_i)\);
2. define how much \((w_i)\);

Mathematically, the problem can be formulated as follows:

3. Minimize: \[ \sum_{i=1}^{n} w_i \sqrt{(x - a_i)^2 + (y - b_i)^2} \]

Over: \((x, y)\)

This problem does not admit a simple algebraic solution. An iterative method is necessary for getting the solution.

3’ Compute the center of gravity \((x, y)\) of \{(\(a_i, b_i\))\};

Each position \((a_i, b_i)\) has a weight \((w_i)\). The solution is thus the center of gravity of this set of points which could be computed by the following physical model. Take a solid map. At each location: perforate the map, introduce a thread in the hole with a weight \((w_i)\) below the map. Tie all the threads together above the map. Hold the map and let all the threads free. The center of gravity should be given by the position of the node.

Another approach consists in using the rectilinear distance:

3’’ Minimize: \[ \sum_{i=1}^{n} w_i \left( |x - a_i| + |y - b_i| \right) \]

Over: \((x, y)\)

Solution: the “median”

\[
\begin{align*}
\sum_{i \mid a_i < x} w_i &= \sum_{i \mid a_i > x} w_i \\
\sum_{i \mid b_i < y} w_i &= \sum_{i \mid b_i > y}
\end{align*}
\]

Let us first consider the case where all the weights are equal to 1. Let assume we have four locations with coordinates: \((1,1)\), \((3,5)\), \((5,2)\) and \((5,4)\). Then, the median of the \(x\) values is any value in \([3,5]\) and the median of the \(y\) values is \([2,4]\). This means that any point in the square \((3,2), (3,4), (5,2), (5,4)\) will minimize the objective function. To convince yourself compute the objective function for the solution \((x,y)=(3,2)\). Then check which distances do change if we move the solution to \((3,2+z)\). In fact, some distances will increase by \(z\) and some will decrease by \(z\). Since we are at the median, there are as many distances that increase as distances that decrease. Globally, the objective function does not change. Similarly you can observe what happens if we leave the median region. In this case, the objective function increases. If a location has a weight of let’s say 2, this is similar to having 2 sites with unit weight at the same location. The solution can be found in the same way. As an example, for the weights 3,4,2 and 2, the median is the point \((5,2)\).
Techniques: 2. Factor-rating systems

The factor-rating method aims at ranking different sites/choices for building a short list.

2.1. **determine factors**

- living conditions
- population
- transportation
- supplies
- taxes

First, you decide which factors are relevant. For this, it is again useful to consider the supply, the production and the delivery processes.

2.2. **weight factors**

Then, relative weights must be given to these factors.

2.3. **rate each site on a same scale**

Then, compute for each possible site, the score obtained for each factor and sum up the results to get the global score.

2.4. **select the site with the highest score**

Here, to illustrate how the method works, we will consider the problem of choosing a study major (the subject in which you will take most of your courses). We assume that you have the choice between 4 main subjects called H, F, M and P.

**Example: Selecting your Major**

It has been assumed that the relevant factors are those of the first column of the next table.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Weight</th>
<th>Score for H</th>
<th>Score for F</th>
<th>Score for M</th>
<th>Score for P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal Interest</td>
<td>2</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Job opportunities</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Amount of work</td>
<td>2</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Success probab.</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47</td>
<td>55</td>
<td>54</td>
<td>41</td>
</tr>
</tbody>
</table>

In this example, each factor has been rated on a same scale (between 0 and 10). Using the factors of column 2, the final score shows that the options M and F are worth further evaluation.

**Technique 3. Detailed cost analysis**

When the set of possible choices has been sufficiently reduced, a detailed cost analysis should be performed.
3. Aggregate Planning

Some questions tackled by strategic planning were:

The strategic planning specifies for each plant:
- the product line;
- the production policy and capacity;
- the process types.

Now that it has been decided (for the long term) which product line is produced at which plant, we can focus on the plant and look inside.

The aggregate planning specifies in a plant:
- the production plan

The production plan specifies how much will be globally manufactured in each time period (usually, a month).

- the workforce and capacity variations

As a result of this global monthly production, the (minor) equipment and the workforce can be planned.

⇒ at the plant level
⇒ for an intermediate term (about 18 months)

This means that a production plan for each product line must be clearly established for the coming months/years (18 months is a usual value). This plan must be feasible in terms of equipment and of workforce and must satisfy the demand. In the general case, different products are manufactured and the following procedure is followed:

Method:

1. define an aggregate unit;

An aggregate unit, such as the labor hour or the machine hour must be selected in order to translate the demand for the different products into the same units. This unit must be related to the capacity you want to plan (machine or manpower).

The choice of an adequate aggregate unit is important and sometimes difficult. It must be as natural as possible and reflect the characteristics which are under study.

2. estimate aggregate demand (over 12-24 months);

Here we need the monthly forecast for all the products for the period considered (the intermediate term). These forecasts are translated into aggregate units.

3. determine an aggregate production plan;

On the basis of this demand, we can select the best production plan.

4. disaggregate the production plan;

The selected production plan is then re-translated into a production plan for the different product lines or groups.

Examples of aggregate units

Choosing an aggregate unit is not easy. The unit should reflect the resource you want to manage. If you want to plan the number of workers, the working hour is a possible choice (although in the next example the radiator kg has been selected).

If you want to plan some critical machine, the machine hour is adequate.

If you want to plan the overall activity, then the expected gross income could be used.
3.1 Example: Radiator Calorix

The company Calorix sells radiators of different sizes: small, medium and large. The demand is very seasonal as shown on the following table. The goal is to plan the production for the coming year and check whether some workers should be hired or not.

Radiators: small - medium - large

The manufacturing of radiators is mainly an assembly task. The selected aggregate unit is the kilogram. In the following Dt summarizes the monthly demand in kilograms assuming that small, medium and large radiators weigh 5, 10 and 25 kilograms respectively. A safety stock St has also been required to face possible demand fluctuations. It has been expressed in kilograms too.

<table>
<thead>
<tr>
<th>Demand</th>
<th>small (units)</th>
<th>medium (units)</th>
<th>large (units)</th>
<th>Dt (kg.)</th>
<th>St (kg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>month</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>200</td>
<td>200</td>
<td>120</td>
<td>6000</td>
<td>3000</td>
</tr>
<tr>
<td>Feb</td>
<td>200</td>
<td>200</td>
<td>40</td>
<td>4000</td>
<td>2500</td>
</tr>
<tr>
<td>Mar</td>
<td>200</td>
<td>100</td>
<td>40</td>
<td>3000</td>
<td>2000</td>
</tr>
<tr>
<td>Apr</td>
<td>380</td>
<td>100</td>
<td>40</td>
<td>3900</td>
<td>2500</td>
</tr>
<tr>
<td>May</td>
<td>400</td>
<td>100</td>
<td>120</td>
<td>6000</td>
<td>3000</td>
</tr>
<tr>
<td>Jun</td>
<td>200</td>
<td>200</td>
<td>240</td>
<td>9000</td>
<td>3500</td>
</tr>
<tr>
<td>Jul</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>11000</td>
<td>4000</td>
</tr>
<tr>
<td>Aug</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>12000</td>
<td>4200</td>
</tr>
<tr>
<td>Sep</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>13000</td>
<td>4400</td>
</tr>
<tr>
<td>Oct</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>12000</td>
<td>4200</td>
</tr>
<tr>
<td>Nov</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>11000</td>
<td>4000</td>
</tr>
<tr>
<td>Dec</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>7000</td>
<td>3000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>97900</td>
<td></td>
</tr>
</tbody>
</table>

The monthly demand and safety stock are represented graphically here below.
Example: data
Before trying to determine the best production plan, let's review the present situation. We assume that 5 workers have already been hired. For the coming months, we require the number of workers to remain between 5 and 10. We have estimated hiring costs of about 50,000 per worker (recruiting cost and training). We also estimated the cost of firing a worker to amount to about 200,000.

• Personal $w_t$:
  $w_0 = 5$ workers
  $5 \leq w_t \leq 10$
  Hiring costs: $H = 50,000$
  Firing costs: $F = 200,000$

The daily production defines the relation between what is planned (the manpower) and the aggregate unit.

• Daily Production $p_t$:
  $p_t \leq 50$ kg per worker and per day

The following costs are needed to differentiate the production costs in regular time, in overtime and by subcontracting.

• Production costs:
  - own raw material 25 / kg
  - regular time 75 / kg
  - overtime 87 / kg
  - subcontracting 150 / kg

External constraints must be specified, if any exist.

• Production constraints:
  - overtime $\leq 0.20$ regular time
  - monthly subcontracting production $\leq 150$ kg

The holding cost corresponds to all the costs which result from the fact that units are not sold immediately but stored temporarily. This cost is expressed per time period. $i_0$ denotes the starting inventory.

• Holding costs and constraints:
  - 30 per kg and per year
  - $i_0 = 2900$ kg

The backlogging cost is difficult to evaluate. This is the penalty we would pay for delaying the delivery without losing the customer.

• Backlogging costs:
  - 10 per kg per month
3.2 Objectives & variables

The central question is how to cope with the seasonal demand.

<table>
<thead>
<tr>
<th>Variable Demand</th>
</tr>
</thead>
</table>

? How to absorb it ?

Alternatives
If you want to produce different volumes in different months, you can use either of the following alternatives:

- **Vary the working time (overtime)**
  - ask the worker to adapt their working time;
- **Vary workforce**
  - fire and hire workers as needed;
- **Vary inventory**
  - build inventory in anticipation of future higher demand;
- **Vary backlogs**
  - ask the customer to wait for their orders;
- **Subcontract**
  - find some external service/production facilities.

<table>
<thead>
<tr>
<th>How to select:</th>
<th>workforce levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>production levels</td>
</tr>
<tr>
<td></td>
<td>inventory levels</td>
</tr>
<tr>
<td></td>
<td>subcontract levels</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>In order to:</th>
<th>meet the demand</th>
</tr>
</thead>
</table>

In order to choose between these different alternatives or the best mix, the costs need first to be listed. These costs are:

**Costs:**
- Basic production costs
- Costs related to workforce changes
- Inventory holding costs
- Backlogging costs

To fulfill the production plan at minimum cost is the first goal. However, do not forget other objectives like flexibility, social peace, worker motivation, ...

**Pure plans:**  
Chase strategy  
Stable workforce - variable work hours  
Level strategy

Different plans are possible, some will be analyzed next.
## 3.3 Graphical Method

For the determination of an aggregate production plan, the graphical method is often used. It provides an overview of what happens. The net demand must be first computed. The net demand is just what is needed in each period.

### 1a. Net Demand:

\[ ND_t = D_t + S_t - S_{t-1} \]

Then we compute the cumulative net demand by summing over the successive periods.

### 1b. Cumulative Net Demand:

\[ \overline{ND_t} = \sum_{i=0}^{t} ND_i \]

The double underlining denotes a cumulative variable.

<table>
<thead>
<tr>
<th>Month</th>
<th>Days</th>
<th>( D_t )</th>
<th>( S_t )</th>
<th>( ND_t )</th>
<th>( \overline{ND_t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>22</td>
<td>6000</td>
<td>3000</td>
<td>6100</td>
<td>6100</td>
</tr>
<tr>
<td>Feb</td>
<td>19</td>
<td>4000</td>
<td>2500</td>
<td>3500</td>
<td>9600</td>
</tr>
<tr>
<td>Mar</td>
<td>21</td>
<td>3000</td>
<td>2000</td>
<td>2500</td>
<td>12100</td>
</tr>
<tr>
<td>Apr</td>
<td>21</td>
<td>3900</td>
<td>2500</td>
<td>4400</td>
<td>16500</td>
</tr>
<tr>
<td>May</td>
<td>22</td>
<td>6000</td>
<td>3000</td>
<td>6500</td>
<td>23000</td>
</tr>
<tr>
<td>Jun</td>
<td>20</td>
<td>9000</td>
<td>3500</td>
<td>9500</td>
<td>32500</td>
</tr>
<tr>
<td>Jul</td>
<td>12</td>
<td>11000</td>
<td>4000</td>
<td>11500</td>
<td>44000</td>
</tr>
<tr>
<td>Aug</td>
<td>22</td>
<td>12000</td>
<td>4200</td>
<td>12200</td>
<td>56200</td>
</tr>
<tr>
<td>Sep</td>
<td>20</td>
<td>13000</td>
<td>4400</td>
<td>13200</td>
<td>69400</td>
</tr>
<tr>
<td>Oct</td>
<td>23</td>
<td>12000</td>
<td>4200</td>
<td>11800</td>
<td>81200</td>
</tr>
<tr>
<td>Nov</td>
<td>19</td>
<td>11000</td>
<td>4000</td>
<td>10800</td>
<td>92000</td>
</tr>
<tr>
<td>Dec</td>
<td>21</td>
<td>7000</td>
<td>3000</td>
<td>6000</td>
<td>98000</td>
</tr>
</tbody>
</table>

The next chart shows the cumulative net demand.
**Graphical Method**

If we decide to work with a fixed number of workers, let us say 9, we can plot the corresponding cumulative production curve.

2a. Plot the production curves for ≠ workforce levels

The cumulative production of a fixed number of workers would be constant if the number of days per month were constant. A way to overcome this problem consists of using the working days on the horizontal axis.

We can try to compute the workforce which would exactly produce the net demand over the chosen horizon. This is given by the formula:

\[
\text{# workers} = \frac{\text{production requested over the horizon}}{\left( \frac{\text{production}}{\text{worker} \cdot \text{day}} \right) \cdot \text{#days in the horizon}}
\]
**Graphical Method**

The above calculation shows that with 8.1 workers, the yearly production could be manufactured in 12 months. Let us now consider the solution with 8 workers.

**2b. Select a workforce and analyze**

The graphical representation in terms of cumulative demand and production implicitly contains additional information. For example, a vertical difference $S$ between the two curves shows the excess stock at some time. At the end of April, for example, the excess stock amounts to 16700 units. The horizontal difference $T$ shows when the corresponding unit (here the last unit produced in April, that is the 33200th unit) has been produced and when it is required (sometime at the beginning of July). It thus shows how long this unit has remained in inventory, unnecessarily.

<table>
<thead>
<tr>
<th>Month $(t)$</th>
<th>Days</th>
<th>$ND_t$</th>
<th>$ND_{t-1}$</th>
<th>$P_t(8)$</th>
<th>$P_{t-1}(8)$</th>
<th>$ES_t = P_t(8) - ND_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>22</td>
<td>6100</td>
<td>6100</td>
<td>8800</td>
<td>8800</td>
<td>2700</td>
</tr>
<tr>
<td>Feb</td>
<td>19</td>
<td>3500</td>
<td>9600</td>
<td>7600</td>
<td>16400</td>
<td>6800</td>
</tr>
<tr>
<td>Mar</td>
<td>21</td>
<td>2500</td>
<td>12100</td>
<td>8400</td>
<td>24800</td>
<td>12700</td>
</tr>
<tr>
<td>Apr</td>
<td>21</td>
<td>4400</td>
<td>16500</td>
<td>8400</td>
<td>33200</td>
<td>16700</td>
</tr>
<tr>
<td>May</td>
<td>22</td>
<td>6500</td>
<td>23000</td>
<td>8800</td>
<td>42000</td>
<td>19000</td>
</tr>
<tr>
<td>Jun</td>
<td>20</td>
<td>9500</td>
<td>32500</td>
<td>8000</td>
<td>50000</td>
<td>17500</td>
</tr>
<tr>
<td>Jul</td>
<td>12</td>
<td>11500</td>
<td>44000</td>
<td>4800</td>
<td>54800</td>
<td>10800</td>
</tr>
<tr>
<td>Aug</td>
<td>22</td>
<td>12200</td>
<td>56200</td>
<td>8800</td>
<td>63600</td>
<td>7400</td>
</tr>
<tr>
<td>Sep</td>
<td>20</td>
<td>13200</td>
<td>69400</td>
<td>8000</td>
<td>71600</td>
<td>2200</td>
</tr>
<tr>
<td>Oct</td>
<td>23</td>
<td>11800</td>
<td>81200</td>
<td>9200</td>
<td>80800</td>
<td>-400</td>
</tr>
<tr>
<td>Nov</td>
<td>19</td>
<td>10800</td>
<td>92000</td>
<td>7600</td>
<td>88400</td>
<td>-3600</td>
</tr>
<tr>
<td>Dec</td>
<td>21</td>
<td>6000</td>
<td>98000</td>
<td>8400</td>
<td>96800</td>
<td>-1200</td>
</tr>
</tbody>
</table>

The above table gives the same data. $P_t$ gives the monthly production and $P_{t-1}$ gives the cumulative values (the number 8 indicates the selected number of workers).
Graphical Method

$ES_t$ in the above table gives the difference between the cumulative production and the cumulative net demand. It shows therefore the evolution of the excess inventory (since the required inventory has been incorporated in the net demand).

2c. Solve the backlog problem

In October, $ES_t$ becomes negative. This means that there will be some backlog. At the end of the period, $ES_t$ amounts to -1200.

backlog = 1200 units

This potential backlog should be solved. Three solutions are possible.

Decision: ? subcontract / overtime / penalty ?

Let us assume here that backlog at the end of the year is not allowed. Additional production capacity is needed. One should determine how it will be provided and when.

? when?

Since overtime is permitted and is cheaper than subcontracting, the former will be chosen. In the following table, $O_t$ denotes the production in overtime and $Q_t$ the cumulative values.

<table>
<thead>
<tr>
<th>Month ($t$)</th>
<th>$ND_t$</th>
<th>$P_t(8)$</th>
<th>Old $ES_t$</th>
<th>$O_t$</th>
<th>$Q_t$</th>
<th>New $ES_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>6100</td>
<td>8800</td>
<td>2700</td>
<td>0</td>
<td>0</td>
<td>2700</td>
</tr>
<tr>
<td>Feb</td>
<td>9600</td>
<td>16400</td>
<td>6800</td>
<td>0</td>
<td>0</td>
<td>6800</td>
</tr>
<tr>
<td>Mar</td>
<td>12100</td>
<td>24800</td>
<td>12700</td>
<td>0</td>
<td>0</td>
<td>12700</td>
</tr>
<tr>
<td>Apr</td>
<td>16500</td>
<td>33200</td>
<td>16700</td>
<td>0</td>
<td>0</td>
<td>16700</td>
</tr>
<tr>
<td>May</td>
<td>23000</td>
<td>42000</td>
<td>19000</td>
<td>0</td>
<td>0</td>
<td>19000</td>
</tr>
<tr>
<td>Jun</td>
<td>32500</td>
<td>50000</td>
<td>17500</td>
<td>0</td>
<td>0</td>
<td>17500</td>
</tr>
<tr>
<td>Jul</td>
<td>44000</td>
<td>54800</td>
<td>10800</td>
<td>0</td>
<td>0</td>
<td>10800</td>
</tr>
<tr>
<td>Aug</td>
<td>56200</td>
<td>63600</td>
<td>7400</td>
<td>0</td>
<td>0</td>
<td>7400</td>
</tr>
<tr>
<td>Sep</td>
<td>69400</td>
<td>71600</td>
<td>2200</td>
<td>0</td>
<td>0</td>
<td>2200</td>
</tr>
<tr>
<td>Oct</td>
<td>81200</td>
<td>80800</td>
<td>-400</td>
<td>400</td>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>Nov</td>
<td>92000</td>
<td>88400</td>
<td>-3600</td>
<td>800</td>
<td>1200</td>
<td>-2400</td>
</tr>
<tr>
<td>Dec</td>
<td>98000</td>
<td>96800</td>
<td>-1200</td>
<td>0</td>
<td>1200</td>
<td>0</td>
</tr>
</tbody>
</table>

1200 units are needed. We shall try to produce them as late as possible. In October, there is a backlog of 400 units. We can satisfy this demand by producing these 400 units in overtime. The remaining 800 units should be produced in November where there is a backlog of 3200 units.

Plan 1 : 8 workers + Overtime in period 10 : 400 kg
in period 11 : 800 kg

The New $ES_t$ values incorporates the production in overtime. Note that a backlog of 2400 units remains in November. This demand will be satisfied in December only.
Graphical Method

We now have a plan without backlog. Let's evaluate its cost.

2d. Evaluate the plan

Plan 1 : 8 workers + Overtime

<table>
<thead>
<tr>
<th>Days</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>cumulated net demand</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>8 workers + overtime</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

Production plan

Production cost

This includes the raw material, the production cost in regular and overtime.

\[ 8 \times 242 \times 50 \times (25 + 75) = 9,680,000 \]
\[ (400 + 800) \times (25 + 87) = 134,400 \]

This includes the hiring and firing costs.

Workforce change cost

\[ 3 \times 50,000 = 150,000 \]

Here, we first need to determine the inventory by summing up the monthly excess inventory ES (when positive).

Inventory holding cost

\[ 95800 \times 30 / 12 = 239,500 \]

Similarly, the backlog is determined by summing up the monthly excess inventories when negative.

Backlogging cost

\[ 2400 \times 10 = 24,000 \]

Subcontracting cost

\[ = 0 \]

Plan 1: cost

\[ = 10,227,900 \]
Graphical Method

Here we consider another plan in which we smoothly increase the number of workers. The idea here is to reduce the large holding cost of plan 1.

3. Consider plans with a variable workforce.

Here we consider another plan in which we smoothly increase the number of workers. The idea here is to reduce the large holding cost of plan 1.

<table>
<thead>
<tr>
<th>Month (t)</th>
<th>$ND_t$</th>
<th>$w_t$</th>
<th>$P_t$</th>
<th>$ES_t$</th>
<th>$O_t$</th>
<th>$Q_t$</th>
<th>New $ES_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>6100</td>
<td>7</td>
<td>7700</td>
<td>1600</td>
<td>0</td>
<td>0</td>
<td>1600</td>
</tr>
<tr>
<td>Feb</td>
<td>9600</td>
<td>7</td>
<td>14350</td>
<td>4750</td>
<td>0</td>
<td>0</td>
<td>4750</td>
</tr>
<tr>
<td>Mar</td>
<td>12100</td>
<td>7</td>
<td>21700</td>
<td>9600</td>
<td>0</td>
<td>0</td>
<td>9600</td>
</tr>
<tr>
<td>Apr</td>
<td>16500</td>
<td>7</td>
<td>29050</td>
<td>12550</td>
<td>0</td>
<td>0</td>
<td>12550</td>
</tr>
<tr>
<td>May</td>
<td>23000</td>
<td>7</td>
<td>36750</td>
<td>13750</td>
<td>0</td>
<td>0</td>
<td>13750</td>
</tr>
<tr>
<td>Jun</td>
<td>32500</td>
<td>8</td>
<td>44750</td>
<td>12250</td>
<td>0</td>
<td>0</td>
<td>12250</td>
</tr>
<tr>
<td>Jul</td>
<td>44000</td>
<td>9</td>
<td>50150</td>
<td>6150</td>
<td>0</td>
<td>0</td>
<td>6150</td>
</tr>
<tr>
<td>Aug</td>
<td>56200</td>
<td>9</td>
<td>60050</td>
<td>3850</td>
<td>0</td>
<td>0</td>
<td>3850</td>
</tr>
<tr>
<td>Sep</td>
<td>69400</td>
<td>9</td>
<td>69050</td>
<td>-350</td>
<td>350</td>
<td>350</td>
<td>0</td>
</tr>
<tr>
<td>Oct</td>
<td>81200</td>
<td>9</td>
<td>79400</td>
<td>-1800</td>
<td>250</td>
<td>600</td>
<td>-1200</td>
</tr>
<tr>
<td>Nov</td>
<td>92000</td>
<td>9</td>
<td>87950</td>
<td>-4050</td>
<td>0</td>
<td>600</td>
<td>-3450</td>
</tr>
<tr>
<td>Dec</td>
<td>98000</td>
<td>9</td>
<td>97400</td>
<td>-600</td>
<td>0</td>
<td>600</td>
<td>0</td>
</tr>
</tbody>
</table>

The decision of increasing the workforce in June and July is arbitrary. Again it can be shown that the production deficit of 600 units at the end of the period is best tackled using overtime in September (350 units) and October (250 units).

**Plan 2 : 7-8-9 workers + Overtime**

- in period 9 : 350 kg
- in period 10 : 250 kg

Production cost

$$97400 \times (25 + 75) = 9.740.000$$

Workforce change cost

$$4 \times 50.000 = 200.000$$

Inventory holding cost

$$64500 \times 30 / 12 = 161.250$$

Backlogging cost

$$4650 \times (10) = 46.500$$

Subcontracting cost

$$= 0$$

**Plan 2 cost**

$$= 10.214.950$$
Graphical Method
Here we will compare the two plans graphically and financially.

4. Compare the different plans

Production plan
![Graphical representation of production plans](image)

- **cost**
The comparison between different plans should not be done on the basis of cost only! But a value is necessary.

- **state at the end of the period**
If different plans keep the company in different states at the end of the considered period, great attention must be given. For example, if one worker must be fired for the next year, this cost must be taken into account. If he is necessary for the next year, on the other hand, this is an advantage.

- **flexibility of the plan**
Here the question is: what if the demand does not behave as expected. Which plan can be more easily modified.

→ **Plan 3 : ...**
Other alternatives could be considered. For example ...
3.4 LP Formulation

The problem of finding the best compromise between workforce changes, inventory, overtime, subcontracting can be formulated as a linear programming problem. Here are the constants we need to formulate the problem.

### Constants and Variables

<table>
<thead>
<tr>
<th>Data</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_H$</td>
<td>cost of hiring one worker</td>
</tr>
<tr>
<td>$c_F$</td>
<td>cost of firing one worker</td>
</tr>
<tr>
<td>$c_I$</td>
<td>cost of holding one unit of stock for one period</td>
</tr>
<tr>
<td>$c_R$</td>
<td>cost of producing one unit on regular time</td>
</tr>
<tr>
<td>$c_O$</td>
<td>incremental cost of producing one unit on overtime</td>
</tr>
<tr>
<td>$c_U$</td>
<td>idle cost per unit of production</td>
</tr>
<tr>
<td>$c_S$</td>
<td>cost to subcontract one unit of production</td>
</tr>
<tr>
<td>$n_t$</td>
<td>number of production days in period $t$</td>
</tr>
<tr>
<td>$K$</td>
<td>number of units produced per day per worker</td>
</tr>
<tr>
<td>$I_0$</td>
<td>initial inventory on hand</td>
</tr>
<tr>
<td>$W_0$</td>
<td>initial workforce</td>
</tr>
<tr>
<td>$D_t$</td>
<td>forecast of demand in period $t$</td>
</tr>
</tbody>
</table>

Here we define more variables than needed. But it makes the formulation simpler.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_t$</td>
<td>number of workers hired in period $t$</td>
</tr>
<tr>
<td>$F_t$</td>
<td>number of workers fired in period $t$</td>
</tr>
<tr>
<td>$I_t$</td>
<td>inventory (in units) in period $t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>production (in units) in period $t$</td>
</tr>
<tr>
<td>$O_t$</td>
<td>production (in units) on overtime in period $t$</td>
</tr>
<tr>
<td>$U_t$</td>
<td>idle time or undertime (in units) in period $t$</td>
</tr>
<tr>
<td>$S_t$</td>
<td>number of units subcontracted in period $t$</td>
</tr>
<tr>
<td>$W_t$</td>
<td>workforce (in workers) in period $t$</td>
</tr>
</tbody>
</table>

Note that the notion of backlog cost has been kept aside for the moment.
LP formulation

The objective function is the minimization of the costs.

**Objective**  
Minimize the total cost

which can be expressed as follows.

\[
\text{Min } \sum_{t=1}^{T} \left( c_H H_t + c_F F_t + c_I I_t + c_R P_t + c_O O_t + c_U U_t + c_S S_t \right)
\]

This includes the costs related to the workforce changes, to the holding of inventories and to the production of units in regular time, in overtime and by subcontractors. Of course, all these variables are related to each other.

Such that:

The number of workers present in each period is defined by the equations:

1. **Conservation of work force**

   \[
   W_t = W_{t-1} + H_t - F_t \quad 1 \leq t \leq T
   \]

   The number of units produced in a period depends on the number of workers and on the use of over and undertime. For example, if all and only the regular time is used for production, then \( P(t) = K n(t) W(t) \), \( O(t)=0 \), \( U(t)=0 \), that is \( W(t) \) workers work \( n(t) \) days and each produce \( K \) units per day. The corresponding cost is \( c_R \) per unit. If some overtime is used, then \( P(t) = K n(t) W(t) + O(t) \) where the \( O(t) \) units are produced in overtime. The total production cost should then be \( K n(t) W(t) \) at cost \( c_R \) and \( O(t) \) units at overtime unit cost. However, the costs are computed differently here. We charge a cost \( c_R \) for the total production \( P(t) \) (in regular and overtime) and an additional cost \( c_O \) for what is produced in overtime. A similar reasoning holds if some idle time is used.

2. **Production equation**

   \[
   P_t = K n_t W_t + O_t - U_t \quad 1 \leq t \leq T
   \]

   The inventory in excess at the end of a period is related to that of the previous period as follows:

3. **Conservation of units**

   \[
   I_t = I_{t-1} + P_t + S_t - D_t \quad 1 \leq t \leq T
   \]

   The nonnegativity of the variables guarantees, for example, that no backlog is allowed.

4. **Non-negativity constraints**

   \[
   H_t, F_t, I_t, O_t, U_t, S_t, W_t, P_t \geq 0 \quad 1 \leq t \leq T
   \]
LP formulation: comments & extension

Here are some comments on this first formulation.

Comments
All the variables are not continuous. The number of workers for example is integer.

1. Integer / continuous variables
   If you decide to round the continuous variable to the next integer, you are not guaranteed to obtain the optimal solution. The LP formulation remains easy only if the number of workers remains constant. Otherwise, this is an integer LP problem.

2. Hiring / firing in the same period
   The problem formulation seems to allow that people get hired and fired in the same period. Fortunately, the variables \( H(t) \) and \( F(t) \) cannot be simultaneously positive in the optimal solution. If it were the case, both variables could be reduced by 1 without changing anything in the constraints. And this would reduce the objective function.

   \[
   \text{the minimization of the objective function}
   \]
   \[
   \text{always guarantees: } H_t = 0 \text{ or } F_t = 0
   \]

   A similar reasoning holds for the variables \( O(t) \) and \( U(t) \).

3. Overtime / undertime in the same period
   \[
   \text{the minimization of the objective function}
   \]
   \[
   \text{always guarantees: } O_t = 0 \text{ or } U_t = 0
   \]

Here we consider different ways of modifying the formulation.

Extensions
Here we first specify a safety stock \( B(t) \) in each period.

1. Inventory constraints
   \[
   I_t \geq 0 \text{ becomes } I_t \geq B_t
   \]

   Here we modify the term of the objective function and the constraints related to the inventory in order to allow some backlog to exist. The variable \( I(t) \) is replaced by the difference between two variables \( I^+ \) and \( I^- \) which cannot be simultaneously positive.

2. Backlogging
   \[
   \begin{cases}
   I_t \geq 0 \\
   c_f I_t
   \end{cases}
   \text{ becomes } \begin{cases}
   I_t = I^+_t - I^-_t \\
   I^+_t, I^-_t \geq 0 \\
   c_f I^+_t + c_B I^-_t
   \end{cases}
   \]
LP formulation: extensions

Here are some other extensions in which a linear function is replaced by a piecewise linear function.

3. Production constraint

\[ P_t \geq 0 \quad 0 \leq P_t \leq P_{\text{max},t} \]

Here, we consider the case of a maximum sustainable production.

4. Work force constraint

Here, we arbitrarily bound the workforce.

\[ W_t \geq 0 \quad W_{\text{min}} \leq W_t \leq W_{\text{max}} \]

In the following example, we assume that the cost of hiring workers is not linear with the number of people hired. Note that the piecewise linear function must remain convex for the linear programming method to work.

Variable (convex piecewise-linear) hiring costs

Let us assume that X workers are hired. If X<H1, then each of these workers generates a hiring cost of C(H1). If (H1 < X < H2), then each of the first H1 workers generates a cost C(H1) and each of the (X-H1) workers left generates a cost C(H2). And so on.

Constraints:

\[
H_t = H_{1t} + H_{2t} + H_{3t}
\]

\[
0 \leq H_{1t} \leq H_1
\]

\[
0 \leq H_{2t} \leq H_2 - H_1
\]

\[
0 \leq H_{3t}
\]

Objective term:

\[
c_{H_1}H_{1t} + c_{H_2}H_{2t} + c_{H_3}H_{3t}
\]

The convexity of the piecewise linear function requires that C(H1) < C(H2) < C(H3). This convexity guarantees the solution of the LP problem to make sense. In order to understand this, let us consider a small example with H1=2, H2=5, C(H1)=1, C(H2)=2, C(H3)=3. If the best solution is to hire 6 workers, then the program will find the solution (H1t=2, H2t=3, H3t=1) with the corresponding cost (2*1 + 3*2 +1*3). The program is not tempted to take the nonsense solution like (H1t=0, H2t=3, H3t=3) since it is more expensive. This would not be the case if the costs were non-convex such as C(H1)=1, C(H2)=2, C(H3)=1.5
3.5 Linear Decision Rule (LDR)

For the solution of the aggregate planning problem, we have already presented two methods: a graphical method and the linear programming approach. Here are two more approaches which are however less often used.

Objective function = Sum of quadratic terms

The idea of the LDR method is to write the objective function as a set of quadratic terms. For example, the production volume $P(t)$ is best set equal to the regular time production. Negative and positive deviations lead to penalties which are here non-linear. By comparison, in the LP formulation, we had a linear term for the overtime and one for the undertime. Proceeding in this way for all the variables, we obtain the following objective function where $c_1$ to $c_6$ are positive constants.

Minimize:

$$
\sum_{t=1}^{T} \left[ c_1 W_t + c_2 (W_t - W_{t-1})^2 + c_3 (P_t - KnW_t)^2 + c_4 P_t + c_5 (I_t - c_6)^2 \right]
$$

Over: $W_t, I_t, P_t \quad 1 \leq t \leq T$

In this formulation, the only real variables are the workforce and the production level since the inventory is tied to the other variables by the following relation.

such that: $I_t = I_{t-1} + P_t - D_t \quad 1 \leq t \leq T$

The underlying idea is that the solution of a quadratic problem is given by a set of linear equations. However, this set of equations does not admit an obvious solution.

Principle:

<table>
<thead>
<tr>
<th>Quadratic objective function</th>
<th>$\downarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear decision rule</td>
<td></td>
</tr>
</tbody>
</table>

Solving the minimization problem leads to a solution of the following type.

For example

$$P_t = \sum_{n=0}^{K} a_n D_{t+n} + b W_{t-1} + c I_{t-1} + d$$

Besides the constant, the first term aims at looking at what is needed in the future while the second and third terms represent a heritage from the past.

Drawbacks: symmetric cost functions

The main weakness of the method is that it requires symmetric cost functions and there is no convincing argument to justify such cost curves.
3.6 Modeling Management Behavior

This last method shows that the intuitive decision a good manager will take is similar to that which is provided by the linear decision rule.

Here are the data.

! \( D_1, D_2, ..., D_t, ..., D_T, \quad I_{nom} \)

Next, here are the variables.

? \( P_1, P_2, ..., P_t, ..., P_T \)

Principle: follow the behavior of a manager!

The first logical decision is to produce in a given period what is needed in this period.

1. Produce what is required

\[
P_t = D_t \quad 1 \leq t \leq T
\]

However, this could result in large changes in production level and workforce.

2. Smooth production over time

It could therefore be useful to introduce smoothing factor \( \alpha \). It is decided to select a production level \( P(t) \) which is a compromise between the current demand and the last production level.

\[
P_t = (1 - \alpha)D_t + \alpha P_{t-1} = D_t + \alpha(P_{t-1} - D_t)
\]

The next step is to take into account the excess inventory (or the backlog) of the previous period.

3. Reach target inventory

We introduce therefore an additional factor \( \beta \).

\[
P_t = D_t + \alpha(P_{t-1} - D_t) + \beta(I_{nom} - I_{t-1})
\]

Finally, a look at the future demands could avoid problems in the future.

4. Incorporate demand forecasts

\[
P_t = \sum_{n=0}^{K} a_n D_{t+n} + \alpha(P_{t-1} - D_t) + \beta(I_{nom} - I_{t-1})
\]

The result is very similar to what the LDR proposed.

The main drawback of the approach is the arbitrary character of all the choices.
3.7 Conclusions

Method:
1. define aggregate unit
2. estimate demand
3. plan (graphically or using LP, LDR, ...)
4. disaggregate

Strengths:
It does only require aggregate data and not a full list of detailed forecasts.

simple, few data
Because different forecasts are summed up, the prediction becomes more accurate.

(more) accurate forecast
Finally, it gives you a global idea of what is going on in your company.

big picture
Here are some points which reduce the use of the approach.

Weaknesses:
need for aggregate units
need for defining costs for aggregate units
fixed time horizon
flexibility not modeled
corporate strategy not modeled
no confidence / trust

Comparison of the planning methods:
Here is a rough comparison of the assumptions necessary for each of the planning method
with some performance indications.

Graphical: no assumption, not optimal, simple, easy to understand;

LP: assumes linear costs, optimal if the workforce remains constant;

LDR: assumes quadratic costs, complicated, optimal;

Management behavior: assume manager are good, non-optimal;
### 4. Disaggregating the Plan: MPS

The aggregate plan gives you the big picture for the medium-term. You know how much you will manufacture month after month. For example, if Plan 1 is selected, we have:

<table>
<thead>
<tr>
<th>month $(t)$</th>
<th>S 5 kg</th>
<th>M 10 kg</th>
<th>L 25 kg</th>
<th>$ND_t$</th>
<th>$P_{t}(8)+O_t$</th>
<th>New $ES_t$</th>
<th>S 5 kg</th>
<th>M 10 kg</th>
<th>L 25 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>200</td>
<td>200</td>
<td>120</td>
<td>6100</td>
<td>8800</td>
<td>2700</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Feb</td>
<td>200</td>
<td>200</td>
<td>40</td>
<td>9600</td>
<td>16400</td>
<td>6800</td>
<td>?</td>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td>Mar</td>
<td>200</td>
<td>100</td>
<td>40</td>
<td>12100</td>
<td>24800</td>
<td>12700</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Apr</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The next step is to translate this plan into a production plan for the coming weeks.

### 4.1 Scope

**Aggregate plan (aggregated unit, month)**

\[ \downarrow \]

**Master Production Schedule (product lines, week)**

In other words, the selected monthly aggregate production must be split into weekly production schedules for the different products. The disaggregation occurs both along the time dimension (shorter terms) and along the product dimension.

**Disaggregation rules:**

1. **Follow the aggregation rule to disaggregate;**
2. **Try to minimize the setups;**
3. **Try to stabilize the production;**
4. **Look at short term data (demand, workforce)**

Different (and contradictory) objectives can be utilized during this disaggregation.
4.2 Examples

Let us now look at an example for the above plan and for the following calendar.

Calendar

<table>
<thead>
<tr>
<th></th>
<th>January (22)</th>
<th>February (19)</th>
<th>March (21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w1</td>
<td>7 14 21 28 4 11 18 25 4 11 18 25</td>
<td>19 25 20 27</td>
<td>w10 w11 w12 w13</td>
</tr>
<tr>
<td>w2</td>
<td>2 9 16 23 30 6 13 20 27 6 13 20 27</td>
<td>7 14 21 28</td>
<td>28 7 14 21 28</td>
</tr>
<tr>
<td>w3</td>
<td>10 17 24 31 17 21 28 7 14 21 28</td>
<td>5 12 19 26 5 12 19 26</td>
<td>28 7 14 21 28</td>
</tr>
<tr>
<td>w4</td>
<td>11 18 25 8 15 22 18 25 4 11 18 25</td>
<td>9 16 23 23 9 16 23 23</td>
<td>28 7 14 21 28</td>
</tr>
<tr>
<td>w5</td>
<td>12 19 26 2 9 16 23 23 9 16 23 23</td>
<td>10 17 24 24 10 17 24 24</td>
<td>28 7 14 21 28</td>
</tr>
<tr>
<td>w6</td>
<td>13 20 22 3 10 17 24 24 10 17 24 24</td>
<td>28 7 14 21 28</td>
<td>28 7 14 21 28</td>
</tr>
</tbody>
</table>

The non-working days have been struck through. As planned, January has 22 working days; February, 19; and March, 21. Remember also the daily production capacity of 400Kg that corresponds to our 8 workers.

Let us first try to find a possible master production schedule for these 13 weeks (3 months) which tries to reduce the number of setups. In this example, we assume that the setup represents some money and not production time. For example, all the setups are performed outside the normal working hours and paid separately.

The customer demand for January is 1000 Kg of small radiators, 2000 Kg of medium radiators, 3000Kg of large radiators and 100Kg of additional safety stock. Altogether, this makes a foreseen customer demand of 6000Kg and a demand for safety stock increase of 100Kg. In addition, an excess production of 2700 Kg has been planned. If we assume that the increase in safety stock is left free (nobody tells us, for example, what kind of radiator should be manufactured), then we are left with a production capacity surplus of 2800 Kg that we can allocate freely.

**Objectives:**
- Meet the demand
- Reduce the number of changes

We could, for example, decide to allocate this surplus of capacity to the manufacturing of large radiators only. In our case, producing 112 additional large radiators (2800 Kg) covers more than the first three months of customer demand.

Concentrating on one product can help avoiding manufacturing this product in the future and therefore save setups.

The questions for January are thus: on which product shall we concentrate the exceeding production capacity and in which order should the products be manufactured?

Below, we propose an answer to these questions and a production plan for the first 9 weeks. Please, as an exercise, try to verify that the solution is feasible and to understand whether it makes sense.

<table>
<thead>
<tr>
<th>month (t)</th>
<th>S 5kg</th>
<th>M 10kg</th>
<th>L 25kg</th>
<th>NDt</th>
<th>Pr(8)+Qt</th>
<th>New ES</th>
<th>S 5 kg</th>
<th>M 10 kg</th>
<th>L 25 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
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<td>2700</td>
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<td>?</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>2000</td>
<td>3000</td>
<td>100</td>
<td></td>
<td>2700</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Minimizing the number of setups can help reduce costs and increase learning. However, it has several drawbacks such as having to manage large and inadequate inventories and becoming
inflexible. Another drawback is the fact that changes (setup) become unusual and therefore not well managed.

The opposite strategy consists in implementing a level schedule.

**Objectives:**

- **Meet the demand**
- **Keep a level schedule**

Such a master production schedule aims at keeping the same production pace, week after week. This plan we propose below has been obtained by choosing the same mix of products, every week. The mix has been determined from the customer demand in the first three months.

<table>
<thead>
<tr>
<th>month ((t))</th>
<th>S 5kg</th>
<th>M 10kg</th>
<th>L 25kg</th>
<th>ND(_t)</th>
<th>(P_r(8)+O_t)</th>
<th>New ES(_t)</th>
<th>S 5 kg</th>
<th>M 10 kg</th>
<th>L 25 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
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<td>40</td>
<td>12100</td>
<td>24800</td>
<td>12700</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

The demand over the first three months is: (small - medium - large) = (600 - 500 - 200) in units. A level schedule would consist in producing in a cyclic way: 6 small radiators, 5 medium and 2 large ones. The production of a single mix (6-5-2) requires 6*5+5*10+2*25=130 units of capacity, here the Kg. The capacity will thus be distributed according to the keys: 6 small radiators per 130 kg, 5 medium radiators per 130 kg and 2 large radiators per 130 kg. Here is the resulting MPS.

<table>
<thead>
<tr>
<th>MPS: level plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
</tr>
<tr>
<td>w1</td>
</tr>
<tr>
<td>Sm</td>
</tr>
<tr>
<td>Md</td>
</tr>
<tr>
<td>Lg</td>
</tr>
</tbody>
</table>

Of course, we should check whether such a plan satisfies the customer demand, month after month.

Again, as an exercise, try to point out the advantages and the drawbacks of such an approach.